# A GENERAL MODEL OF LEGGED LOCOMOTION ON NATURAL TERRAIN

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# A GENERAL MODEL OF LEGGED LOCOMOTION ON NATURAL TERRAIN


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# Foreword

Dynamic modeling is the fundamental building block for mechanism analysis, design, control and performance evaluation. One class of mechanism, legged machines, have multiple closed-chains established through intermittent ground contacts. Further, walking on natural terrain introduces non-linear system compliance in the forms of foot sinkage and slippage. Closed-chains constrain the possible motions of a mechanism while compliances affect the redistribution of forces throughout the system.

This research monograph, which is based on David Manko's dissertation research at Carnegie Mellon University, develops a dynamic mechanism model that characterizes indeterminate interactions of a closed-chain robot with its environment. The approach is applicable to any closed-chain mechanism with sufficient contact compliance although legged locomotion on natural terrain is chosen to illustrate the methodology. The modeling and solution procedures are general to all walking machine configurations, including bipeds, quadrupeds, beam-walkers and hopping machines.

The general approach is to identify and model the contact compliance, and then define a set of generalized coordinates that are advantageous for modeling the robot. Equations of motion are formulated which are applicable to all walker configurations through suitable definition of geometric parameters. The nature of the ground compliance is determined empirically from experimentation, and related to existing terramechanics models. Novel solution procedures are developed specifically for this class of problem that accommodate all combinations of imposed boundary conditions and these procedures have proven to be extremely robust.

This work develops a functional model of legged locomotion that incorporates, for the first time, non-conservative foot-soil interactions in a non-linear dynamic formulation. The model was applied to a prototype walking machine, and simulations generated significant insights into walking machine performance on natural terrain. The simulations are original and essential contributions to the design, evaluation and control of these complex robot systems. While posed in the context of walking machines, the approach has wider applicability to rolling locomotors, cooperating manipulators, multi-fingered hands, and prehensile agents. I believe that this monograph provides methods, insights and is a reference that will be useful to researchers in these areas.

Controlled closed-chain devices are emerging as an important class of capable mechanisms. Walkers, hands and multiple arms, which have been discounted for their complexity and control difficulties, are now maturing into viable robot forms. This work is a milestone toward the understanding and realization of these important robots forms of our future.

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# Chapter 1

# Introduction

Walking machines have the potential to navigate terrains that are inaccessible to wheeled locomotors. Advantages of legged locomotion include terrain-isolated propulsion, reduced soil work, a stable work platform, smooth body trajectories, and adaptability of gaits for obstacle avoidance. However, existing walking machines are research prototypes limited to navigating smooth, non-compliant surfaces. There is virtually no experience with autonomous, integrated systems that traverse rugged terrain featuring varying degrees of compliance. Walking on natural terrain imposes transitional foot contact loads, foot sinkage and slippage, and foot placements on rocks, uneven surfaces and slopes.

Research in this area requires a functional model of legged locomotion on natural terrain to be used for simulation studies, performance evaluations, and model-based control. Applications include development of mechanical configurations that minimize power consumption, maximize stability, and increase payload-to-weight ratio. In addition, advances in perception, planning, and fail-safe control are needed for fully autonomous operation. Gait selection and footfall planning are unique to legged locomotion and require significant modeling before legged mobility can be fully capable in unstructured terrains.

## 1.1 Motivation

Sufficient analysis of a walking mechanism is required to guarantee that power and performance goals can be attained because challenges in previous development of physical walkers - immature components, integration complexity, cost, scale and fabrication issues - limited the possible extent of earlier experimental investigation. A model of legged locomotion on natural terrain is required to develop control algorithms, gait planning and selection, and performance criteria for power consumption, stability margins and maximum grade of traversal. Estimates of mechanism force distributions during operation, which are necessary for mechanical design, can be established through model simulations. Also, control algorithms can be formulated and tested in a timely and less committing manner through model simulations compared to direct development on robotic hardware.

Locomotors are gravity stabilized and have a potential for tipover, which could be disastrous to a mission. On rough terrain, an autonomous walking machine must plan a path to the desired destination, while considering the potential for tipover during traversal. A functional model of legged locomotion on natural terrain is useful for proactive assessment of candidate paths and gait selections considering machine stability and power consumption.

Many walking machine configurations could conceivably operate on natural terrain, and it is desirable that the formulated legged locomotion model be sufficiently general to all designs. Also, successful simulations depend on the development of solution procedures which are appropriate for all possible applications of the model. A model of legged locomotion on natural terrain must apply to extreme events, such as accidental foot contacts, with fidelity and robustness that is transparent to a community of users from designers to power engineers to control scientists.

## 1.2 Problem Statement

Development of a functional model is essential for realization of legged locomotion on natural terrain. Although most walking machines operate with relatively small accelerations and, therefore, experience minimal inertial loadings, compliances in the robot and environment can produce significant time-dependent mechanism responses that must be evaluated with a dynamic formulation. Further, compliance in a closed-chain mechanism (e.g., a walking machine on compliant terrain) serves to redistribute forces throughout the system, a phenomenon which must be modeled in detail for the design and control of these systems. Existing legged locomotion models have not incorporated system compliance, particularly foot-soil interactions, in a full dynamic formulation.

A model of legged locomotion on natural terrain is formulated in this work that incorporates non-linear foot-soil interactions into a full dynamic formulation. Otherwise, link members and joints are considered to be ideally rigid, allowing a tractable number of degrees of freedom (dofs). Appropriate joint damping and backdriving models are identified through experimentation. Novel solution procedures are devised for this class of problem which are extremely robust and adaptable to all problem variations arising from different combinations of imposed boundary conditions. The modeling and solution procedures are general to all walking machine configurations including bipeds, quadrupeds, beam-walkers, and hopping machines.

## 1.3 Monograph Overview

Chapter 2 presents background relevant to modeling legged locomotion on natural terrain; existing models of legged locomotion, dynamic formulations, constraint modeling, representations of foot-soil interactions, and joint modeling are discussed. Formulation of the legged locomotion model including closed-chain mechanism dynamics, ground interactions and joint behavior is described in Chapter 3. Procedures are presented in Chapter 4 which efficiently calculate stable, accurate solutions in a novel manner, once the model has been formulated.

The legged locomotion model is applied to a prototype walking machine in Chapter 5 and simulations described in Chapter 6 are used to verify the modeling and solution procedures. The utility of the model is demonstrated through gait cycle and leveling control simulations of the prototype walking machine and model generality is illustrated by application of the model to an alternate configuration (Chapter 7). A summary of this monograph is provided in Chapter 8. The appendices detail foot-soil testing, numerical solution parameters, and development of a simplified massless leg model.

# Chapter 2

# Background

This monograph develops a model of legged locomotion on natural terrain that, for the first time, incorporates non-conservative foot-soil interactions into a full dynamic formulation. Hence, methods for deriving the equations of motion for robotic systems are essential building blocks. Further, legged machines have multiple closed-chains[1] established through ground contacts that must be represented in the dynamic formulation; approaches for modeling motion constraints are examined.

Walking imposes transitional foot contact loads, sinkage and slippage of feet, and foot placements on rocks, uneven terrain, and slopes. Therefore, characterization of foot contact in natural terrain is essential for prediction and control of off-road walking machines. Joint frictional and backdriving effects increase power consumption and can be problematic to a walker's control system. In order for the legged locomotion model to be useful for control system simulations and performance evaluations, joint modeling must adequately predict these effects. This chapter presents background relevant to these phenomena, which are essential to the modeling of legged locomotion on natural terrain.

## 2.1 Existing Models of Legged Locomotion

The earliest models of biped locomotion [1], [2], [3] only considered dynamics of the body while neglecting leg inertias and mechanism compliance. The formulation of [4] included leg inertias but system compliance was not addressed. These biped locomotion models were useful for calculating the mechanism response to a specified history of applied forces (i.e., forward dynamic model).

The non-compliant legged locomotion models in [5], [6] idealize a walking machine initially without ground contacts, then restore contact conditions in the course of analysis. Thus, the mechanism is transformed into a set of serial

[1] Closed path (i.e., a curve beginning and ending at the same point) in a structure around which forces are transmitted.

link manipulators with a common base (i.e., the body) for which the recursive Newton-Euler algorithm is applicable to each chain (or leg). The equations of motion are derived by considering the dynamic equations for each leg, equilibrium of the body, and constraint equations for each closed-chain.

The resulting system equations comprise a set of simultaneous linear algebraic equations, where the unknown quantities for the forward dynamic model [5] are the joint accelerations, body accelerations and constraint forces. In this formulation, the unknown accelerations and forces are assumed to remain constant during a timestep; displacements and velocities are calculated by explicit integration of the accelerations. The stepsizes required for an accurate solution are small and, therefore, more numerous than the stepsizes that are possible with implicit procedures.

Joint forces required to produce desired mechanism motions are calculated using inverse dynamic models; these models are obtained by substituting joint trajectories into the equations of motion. The inverse dynamic equations for a non-compliant, closed-chain mechanism are the same as the forward dynamic equations except that the constraint equations are no longer applicable because the specified joint motions identically satisfy the kinematic constraints. The unknown quantities for inverse dynamic models of non-compliant, legged locomotion [6] are the joint forces and constraint forces; these variables are underspecified since the constraint equations have been eliminated. Linear programming techniques were used to calculate optimized solutions that minimize input power while satisfying maximum limits on joint and traction forces.

Models of legged locomotion with compliant joints represented by triaxial springs are derived in [7]; a set of non-generalized coordinates were used to formulate the model without consideration of closed-chains established through ground contacts. The resulting model predicts kinematically inadmissible motions which violate the constraint equations that are required to enforce an admissible mechanism response. Vertical foot-soil interactions have been included in a static legged locomotion model of a hexapod walker with an alternating tripod gait [8], but this model is inappropriate for the indeterminant condition when more than three legs contact the ground.

## 2.2 Dynamic Formulation

Different methods for deriving the equations of motion for robotic systems are examined here in the context of walking machines. The discussion is limited to those methods that are applicable to rigid mechanisms (i.e., rigid members and joints) with compliant contacts because the legged locomotion model formulated in this work considers the mechanism to be rigid (with an option for lumping mechanism compliance at the contact points). Legged locomotion is characterized by multiple closed-chains established through ground contacts that affect the resulting mechanism motions. Approaches for modeling the constraints of closed-chain mechanisms are discussed.

### 2.2.1 Mechanism Dynamics

**Lagrangian Dynamics** Lagrange's equations of motion are specified as functions of the potential and kinetic energies of a body. As a result, interbody constraint forces do not require consideration because these forces produce no useful work (i.e., do not add to the energy of the system). The use of scalar energy expressions combined with eliminating the need to consider constraint forces permits relatively simple application of Lagrangian dynamics to complex configurations when compared to other methods that use vector quantities. Application of Lagrange's equations of motion to robotic systems using $(4 \times 4)$ homogeneous transformation matrices (for specifying the mechanism geometry) represents a formal method of formulating dynamic equations [31]; the derivation of this approach is commonplace and will not be reiterated here.

Lagrangian dynamics is shown in [9] to be inefficient relative to the number of additions and multiplications required [9] for inverse dynamic calculations. A recursive Lagrangian dynamic formulation is presented in that reference where the coefficients for the dynamic equations are calculated recursively, resulting in a reduced computational dependency. Mechanism closed-chains can be represented in a Lagrangian formulation with constraint equations and Lagrange multipliers (or equivalent constraint forces), or with a reduced set of generalized coordinates.

**Newton-Euler Dynamics** Alternately, the equations of motion for a mechanism can be derived by applying the Newton-Euler equations to each link [32]. The Newton-Euler equations account for all forces and moments acting on a link including constraint forces between links and any applied forces (i.e., joint forces, contact loadings, etc.). The system equations are obtained in any suitable inertial frame by assembling the individual link equations while eliminating constraint forces between links.

The Newton-Euler formulation requires consideration of force vectors (i.e., magnitude and direction), which combined with the need to eliminate constraint forces, makes this approach more difficult to apply to complex configurations compared with other methods that consider only scalar quantities (e.g., work or energy). Constraint forces resulting from closed-chains are transmitted through a mechanism affecting the interlink constraint forces. Since constraint forces between links require explicit consideration with the Newton-Euler formulation, closed-chains must be represented with constraint equations and Lagrange multipliers (or equivalent constraint forces).

A recursive Newton-Euler algorithm [10] is available for the efficient calculation of inverse dynamic solutions for tree structured mechanisms. The recursive approach calculates link velocities from the base to distal links and then calculates link forces from the tip (or end-effector) to the base. The joint forces required to produce the desired mechanism trajectory are obtained from the calculated link forces.

**D'Alembert's Principle** D'Alembert's Principle can be used to derive the equations of motion for a robotic system by defining all forces acting on a body which is undergoing an acceleration; the vector sum of forces results in equilibrium equations for a member. The link equilibrium equations are simultaneously solved, consistent with the boundary conditions, to obtain the dynamic response of the system. Closed-chains must be represented with constraint equations and Lagrange multipliers (or equivalent constraint forces) because interlink constraint forces must be explicitly defined.

Definition of all forces acting on a member is difficult because centrifugal and coriolis forces are not easily envisioned for inclusion in the equilibrium equations. As a result, centrifugal and coriolis effects are usually neglected when using D'Alembert's principle which limits application of the method to mechanisms that operate at relatively low speeds. Use of force vectors makes this approach more difficult to apply relative to other methods that consider only scalar quantities (e.g., work or energy).

**Kane's Dynamics** Kane's dynamical equations are yet another method used for formulation of the equations of motion for robotic systems [11]. Kane's dynamics is a subset of the more general class of methods known as Lagrange's form of D'Alembert's principle (or Lagrange's Principle) [12]. The essence of Kane's dynamics is to multiply the Newton-Euler equations with selected vectors (called partial velocities and partial angular velocities, which are non-dimensional quantities) to obtain scalar representations of the forces acting on a body. As a result of the dot product operation, interbody constraint forces are shown [49] to cancel during assembly of the dynamic equations and do not require consideration.

A set of generalized speeds is defined for a configuration and the linear and angular velocities of each member determine the partial velocity and partial angular velocity vectors used to multiply the Newton-Euler equations. Generalized speeds can be expressed as functions of the joint velocities, which is the same form as non-holonomic constraint equations (i.e., functions of joint velocities) representing closed-chains in a mechanism. As a result, a dependent set of generalized speeds is amenable to coordinate reduction through Gaussian elimination into an independent set of variables [33]. Therefore, constraint equations can be represented in Kane's dynamics through definition of an independent set of generalized speeds [13], Gaussian elimination of a dependent set of generalized speeds, or Lagrange multipliers and constraint equations.

A judicious choice of generalized speeds can simplify the resulting equations of motion as shown by the application of Kane's dynamics to the Stanford Arm [11]. Other advantages of this approach include the relative ease of scalar operations and direct accommodation of closed-chains in a mechanism by a suitable definition of generalized speeds. Application of Kane's dynamics requires extensive symbolic manipulations and the appropriate choice of generalized speeds is not always apparent, which are disadvantages of the method. Also, the resulting model is suitable only for the mechanism configuration under consideration.

### 2.2.2 Constraint Modeling

Closed-chains in a mechanism constrain the possible motions of that system. Holonomic constraints that characterize fixed-base manipulators can be expressed as functions of position variables while non-holonomic constraints that apply to wheeled mobile robots can only be defined in terms of velocities; constraint type can be determined as shown in [14]. Legged locomotion is characterized by holonomic constraints because the associated constraint equations can be expressed as functions of the joint positions [15]. The type of constraint can affect the choice of solution technique [34]; simpler conditions (and procedures) are associated with holonomically constrained systems.

Closed-chain mechanisms are most effectively modeled with generalized coordinates that produce a minimum number of dynamic equations from the outset. This approach eliminates the need to generate, then condense excess equations. A valid set of generalized coordinates must be an independent set of variables that represent all kinematically admissible mechanism motions [35]. Therefore, all possible mechanism motions can be uniquely expressed as functions of the generalized coordinates. General guidelines are not available for identifying sets of generalized coordinates because each configuration may require a different set of coordinates. The choice of coordinates is based primarily on the analyst's experience and insight.

Lagrange multipliers are commonly used for incorporating constraints into the equations of motion [16] for a mechanical system. The constraint equations (specified as velocity functions) are multiplied by Lagrange multipliers and the coefficients of each velocity variable are appended to the dynamic equation for that variable. (These additional terms represent constraint forces acting on the corresponding dof.) The equations of motion are composed of dynamic and constraint equations where the unknowns include the Lagrange multipliers in addition to the dependent set of variables.

Advantages of using Lagrange multipliers are that the method is formalized, suitable for all applications, and constraint forces are explicitly calculated. Disadvantages of the approach are the increased number of variables and equations compared to other methods that reduce the dependent set of variables. Also, equations of motion that have been formulated with Lagrange multipliers are a singular set of differential equations that require special solution procedures [36].

The remaining methods for incorporating constraints into the equations of motion involve reducing the dependent set of variables consistent with the constraint equations. The simplest approach is to apply Gaussian elimination to solve for a subset of coordinates as functions of the remaining variables. These expressions are back-substituted into the dynamic equations, thus reducing the set of variables. Gaussian elimination of position variables is possible only in relatively simple cases and solution singularities can occur because of constraint changes or the choice of eliminated coordinates [18].

More general approaches for reducing the original set of coordinates use orthogonal transformations characterized by the eigenvalues and eigenvectors of the constraint equations. The zero-eigenvalues theorem [17] defines a transformation matrix as a collection of independent eigenvectors associated with the zero eigenvalues of $A^TA$ where $A$ is the coefficient matrix of the constraint equations. Another orthogonal transformation method results from application of singular value decomposition to the coefficient matrix [18] for definition of the transformation matrix from original to reduced coordinates. Coordinate reductions through orthogonal transformations are formal procedures that avoid the solution singularities associated with Gaussian elimination. The disadvantage of using these reduction techniques is the computational complexity of the required operations.

## 2.3 Foot-Soil Interactions

Soil loading characteristics of a walking machine foot pad are similar to the those observed during plate testing on soil. Walking machines are relatively slow moving (existing machines have a top speed of $8\,mph$) and do not propel soil mass under normal operation so that soil dynamic effects can be neglected in the representation of foot-soil interactions. Soil response to foot placement is fast relative to foundation settlement, which eliminates the need to consider consolidation due to reduction of pore water pressure over time.

Modeling foot-soil interactions requires force-deflection relationships for different loading and slope conditions of foot placement. The bulk of relevant technology is located in the terramechanics literature which is a discipline that predicts vehicle performance on natural terrain. A fundamental approach in terramechanics is to determine (experimentally or analytically) force-deflection characteristics of a representative soil sample loaded by plates of various sizes and shapes. This information is then extrapolated to predict vehicle performance.

Available force-deflection relationships for modeling vertical and combined vertical-lateral plate loading conditions are discussed below. No available references addressed the special cases such as rotational loading of a plate on soil under various combinations of vertical/lateral loads, placement on sloped surfaces, or penetration of variably oriented plates into soils. Significant research efforts are required to completely understand the complex phenomena of foot-soil interactions and the effect of foot design parameters such as sole composition and shape.

### 2.3.1 Vertical Force-Deflection Relationships

Empirical force-deflection relationships are used to characterize vertical plate loading on soils. The method is applied by fitting parameters of a chosen functional relationship to experimental load-deflection data. Variations in soil type (e.g., clay, sand) and soil properties with depth (non-homogeneous soils) are characterized by definition of parameters and choice of function types.